27/7

B.Sc. Part I (Hons). 1st Paper

TRIGONOMETRY (CONTD.)

Summation of series (contd.)

1. Sum the series

$$\cos\theta \cdot \sin\theta + \frac{\cos^2\theta \sin 2\theta}{\lfloor 2} + \frac{\cos^3\theta \sin 3\theta}{\lfloor 3} + \cdots \text{ to } \infty$$

Soln.

Let $S = \cos\theta \cdot \sin\theta + \frac{\cos^2\theta \sin 2\theta}{\lfloor 2} + \frac{\cos^3\theta \sin 3\theta}{\lfloor 3} + \cdots \text{to} \infty$

i.e. $S = \sin\theta \cdot \cos\theta + \frac{\sin 2\theta \cdot \cos^2\theta}{\lfloor 2} + \frac{\sin 3\theta \cdot \cos^3\theta}{\lfloor 3} + \cdots \text{to} \infty$

Let $C = 1 + \cos\theta \cdot \cos\theta + \frac{\cos 2\theta \cdot \cos^2\theta}{\lfloor 2} + \frac{\cos 3\theta \cdot \cos^3\theta}{\lfloor 3} + \cdots \text{to} \infty$

$\therefore C + iS = 1 + \cos\theta(\cos\theta + i\sin\theta) + \frac{\cos^2\theta}{\lfloor 2}(\cos 2\theta + i\sin 2\theta)$
$+ \frac{\cos^3\theta}{\lfloor 3}(\cos 3\theta + i\sin 3\theta) + \cdots \text{to} \infty$

$\Rightarrow C + iS = 1 + \cos\theta \cdot e^{i\theta} + \frac{\cos^2\theta}{\lfloor 2} \cdot e^{2i\theta} + \frac{\cos^3\theta}{\lfloor 3} e^{3i\theta} + \cdots \text{to} \infty$

Put $e^{i\theta} \cdot \cos\theta = x$

$\Rightarrow C + iS = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots \text{to} \infty.$

$\Rightarrow C + iS = e^{x} = e^{e^{i\theta} \cdot \cos\theta} = e^{(\cos\theta + i\sin\theta) \cdot \cos\theta}$

$\Rightarrow c+is = e^{\cos^2\theta + i\sin\theta\cos\theta}$

$\Rightarrow c+is = e^{\cos^2\theta} \cdot e^{i\sin\theta\cos\theta}$

$= e^{\cos^2\theta} \cdot e^{i(\sin\theta\cos\theta)} = e^{\cos^2\theta} \cdot \left[\cos(\cos\theta\sin\theta) + i\sin(\cos\theta\sin\theta)\right]$

$\Rightarrow c+is = e^{\cos^2\theta} \cdot \cos(\sin\theta\cos\theta) + i e^{\cos^2\theta} \cdot \sin(\sin\theta\cos\theta)$

Equating imaginary parts, we get

$\Rightarrow s = e^{\cos^2\theta} \cdot \sin(\sin\theta\cos\theta) = e^{\cos^2\theta} \cdot \sin\left(\frac{1}{2}\sin 2\theta\right)$.

2. Sum the series

$$1 + e^{\sin\alpha} \cdot \cos(\cos\alpha) + \frac{e^{2\sin\alpha} \cdot \cos(2\cos\alpha)}{\lfloor 2} + \cdots \text{ to } \infty$$

Soln. Let $c = 1 + e^{\sin\alpha}\cos(\cos\alpha) + \frac{e^{2\sin\alpha} \cdot \cos(2\cos\alpha)}{\lfloor 2} + \cdots \text{ to } \infty$.

and $s = e^{\sin\alpha}\sin(\cos\alpha) + \frac{e^{2\sin\alpha} \cdot \sin(2\cos\alpha)}{\lfloor 2} + \cdots \text{ to } \infty$

$\Rightarrow c+is = 1 + e^{\sin\alpha}\left[\cos(\cos\alpha) + i\sin(\cos\alpha)\right]$

$+ \frac{e^{2\sin\alpha}}{\lfloor 2}\left[\cos(2\cos\alpha) + i\sin[2\cos\alpha]\right] + \cdots \text{ to } \infty$

$$\Rightarrow C + iS = 1 + e^{\sin\alpha} \cdot e^{i\cos\alpha} + \frac{e^{2\sin\alpha}\, e^{i\cdot 2\cos\alpha}}{\lfloor 2} + \dots \text{ to } \infty$$

Put $x = e^{\sin\alpha} \cdot e^{i\cos\alpha} = e^{\sin\alpha + i\cos\alpha}$

$$\Rightarrow C + iS = 1 + x + \frac{x^2}{\lfloor 2} + \dots \text{ to } \infty$$

$$\Rightarrow C + iS = e^{x}$$

$$\Rightarrow C + iS = e^{e^{\sin\alpha + i\cos\alpha}} = e^{e^{\sin\alpha} \cdot e^{i\cos\alpha}}$$

$$\Rightarrow C + iS = e^{e^{\sin\alpha} \cdot [\cos(\cos\alpha) + i\sin(\cos\alpha)]}$$

$$\Rightarrow C + iS = e^{e^{\sin\alpha} \cdot \cos(\cos\alpha) + e^{\sin\alpha} \cdot i\sin(\cos\alpha)}$$

$$= e^{e^{\sin\alpha} \cdot \cos(\cos\alpha) + i\, e^{\sin\alpha} \cdot \sin(\cos\alpha)}$$

$$= e^{e^{\sin\alpha} \cdot \cos(\cos\alpha)} \cdot e^{i\cdot(e^{\sin\alpha} \cdot \sin(\cos\alpha))}$$

$$\Rightarrow C + iS = e^{e^{\sin\alpha} \cdot \cos(\cos\alpha)} \cdot \left[\cos\left(e^{\sin\alpha} \cdot \sin(\cos\alpha)\right) + i\sin\left(e^{\sin\alpha} \cdot \sin(\cos\alpha)\right)\right]$$

Equating real parts, we get

$$C = e^{e^{\sin\alpha} \cdot \cos(\cos\alpha)} \cdot \cos\left(e^{\sin\alpha} \cdot \sin(\cos\alpha)\right)$$

Q. Sum the series

$$\frac{5\cos\theta}{\lfloor 1} + \frac{7\cos 3\theta}{\lfloor 3} + \frac{9\cos 5\theta}{\lfloor 5} + \cdots \text{ to } \infty.$$

Soln Let $C = \frac{5\cos\theta}{\lfloor 1} + \frac{7\cos 3\theta}{\lfloor 3} + \frac{9\cos 5\theta}{\lfloor 5} + \cdots \text{ to } \infty$

and $S = \frac{5\sin\theta}{\lfloor 1} + \frac{7\sin 3\theta}{\lfloor 3} + \frac{9\sin 5\theta}{\lfloor 5} + \cdots \text{ to } \infty$

$$\Rightarrow C + iS = \frac{5(\cos\theta + i\sin\theta)}{\lfloor 1} + \frac{7(\cos 3\theta + i\sin 3\theta)}{\lfloor 3}$$

$$+ \frac{9(\cos 5\theta + i\sin 5\theta)}{\lfloor 5} + \cdots \text{ to } \infty$$

$$\Rightarrow C + iS = 5\frac{e^{i\theta}}{\lfloor 1} + \frac{7e^{3i\theta}}{\lfloor 3} + \frac{9e^{i5\theta}}{\lfloor 5} + \cdots \text{ to } \infty$$

$$\Rightarrow C + iS = 4\left(\frac{e^{i\theta}}{\lfloor 1} + \frac{e^{3i\theta}}{\lfloor 3} + \frac{e^{5i\theta}}{\lfloor 5} + \cdots \text{ to } \infty\right)$$

$$+ \left(\frac{e^{i\theta}}{\lfloor 1} + \frac{3e^{i\theta}}{\lfloor 3} + \frac{5e^{5i\theta}}{\lfloor 5} + \cdots \text{ to } \infty\right)$$

$$\Rightarrow C + iS = 4\sinh(e^{i\theta}) + e^{i\theta}.\cosh(e^{i\theta})$$

$$= \frac{4}{i}\sin(ie^{i\theta}) + e^{i\theta}\cos(ie^{i\theta})$$

$$= -4i\sin\{i(\cos\theta + i\sin\theta)\} + (\cos\theta + i\sin\theta).\cos\{i\cos\theta - \sin\theta\}$$

Expanding $\sin i(\cos\theta + i\sin\theta)$ and separating real parts, we can find the required sum.